Equivalent Angle-of-Attack Method for Estimating Nonlinear Aerodynamics of Missile Fins

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A method has been developed for estimating the nonlinear aerodynamic characteristics of missile wing and control surfaces. The method is based on the assumption that if a fin on a body has the same normal-force coefficient as a wing alone composed of two of the same fins joined together at their root chords, then the other force and moment coefficients of the fin and the wing alone are the same, including the nonlinearities. The method can be used for deflected fins at arbitrary bank angles and at high angles of attack. In this paper a full derivation of the method is given, its accuracy is demonstrated, and its use in extending missile data bases is shown.

Nomenclature

a = body radius
$\mathcal{R}$ = aspect ratio of wing alone
c_R = length of root chord
C_N = normal-force coefficient
$C_{N_{F(B)_i}}$ = normal-force coefficient of fin i in the presence of a circular body
C_{N_W} = normal-force coefficient of the wing alone, which is composed of two fins joined at their root chords
$(C_{N_\alpha})_W$ = slope at zero angle of attack of the normal-force coefficient curve of the wing alone
k_W = fin deflection factor, see Eq. (1)
K_W = Beskin upwash factor, see Eq. (1)
K_ϕ = sideslip factor, see Eq. (7)
M_∞ = freestream Mach number
s = semispan of wing alone
s_m = semispan of fin as measured from the body axis
V_{n_i} = component of average velocity acting normal to fin i
V_{p_i} = component of average velocity acting parallel to root chord of fin i
V_∞ = freestream velocity
$\bar{x}_{F(B)}$ = axial location of center of pressure for a fin in the presence of a body
$\bar{x}_W$ = axial location of center of pressure for wing alone
$\bar{y}_{F(B)}$ = spanwise location of center of pressure of fin in the presence of a body measured from body axis
$\bar{y}_W$ = spanwise location of center of pressure of wing alone
α_C = angle between body axis and wind velocity vector
α_{eq_i} = equivalent angle of attack of fin i, i.e., angle of attack of wing alone which gives same normal-force coefficient as that of fin i
$\hat{\alpha}_{eq_i}$ = equivalent angle of attack of fin i if all fins are undeflected
δ_i = deflection of fin i, positive when the leading edge is rotated toward the leeward side of the body
$(\Delta\alpha_{eq})_{ji}$ = increment in equivalent angle of attack of fin i due to deflection of fin j
$(\Delta\alpha)_{v_i}$ = average angle of attack induced on fin i by vortices
Λ_{ji} = fin deflection factor, see Eqs. (11) and (12)
ϕ_B = bank angle of wing-body combination, see Fig. 3
ϕ_i = roll angle of fin i, see Fig. 3

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Introduction

COMPREHENSIVE predictions of the aerodynamic characteristics of tactical missiles require estimations of the component forces and moments. Until recently, estimating fin (or wing) forces and moments in the presence of a body and other fins with sufficient accuracy to predict lateral and control characteristics up to high angles of attack was rarely attempted for configurations without a previously developed data base. Now, with the advent of systematic data bases[1-4] and the continued development of vortex tracking methods,[5-8] the task is considerably easier. However, some means are still necessary for properly accounting for effects not in the data base, e.g., different span-to-body diameter ratios and different vortical flowfields. The methods we have developed for this purpose depend upon the equivalent angle-of-attack (α_{eq}) concept which was introduced in Ref. 5 and expanded in Ref. 6. The purpose of this paper is to develop the α_{eq} idea in detail and demonstrate its usefulness.

In the following sections, we introduce the α_{eq} concept for the small-angle approximation and extend it to the nonlinear range. We show how to extend the concept to very high angles of attack, and we give several examples of how the concept can be used to extend available data bases.

Introduction to the α_{eq} Concept

The equivalent angle-of-attack (α_{eq}) method is based on the assumption that if a fin on a body has the same normal-force coefficient (based on planform area) as a wing alone composed of two of the same fins joined together at their root chords, then the other force and moment coefficients of the fin and the wing alone are the same, including the nonlinearities. The method can be easily incorporated into existing missile aerodynamics computer programs and is suitable for hand calculation.

The genesis of the present approach is found in the small-angle "modified" slender-body theory of Ref. 9. Using those results, we can show that the normal-force coefficient acting on the equally deflected horizontal fins of a missile in the "plus" attitude is given by

$$C_{N_{F(B)}} = [K_W \alpha_C + k_W \delta + (\Delta\alpha)_v] (C_{N_\alpha})_W \tag{1}$$

where $C_{N_{F(B)}}$ is the normal-force coefficient acting on the fins in the presence of the body, K_W the Beskin upwash factor, k_W a factor which accounts for the nonperfect reflection plane at the fin root, $(\Delta\alpha)_v$ an average angle of attack induced on the fins by vortices, and $(C_{N_\alpha})_W$ the slope at zero angle of attack of the normal-force coefficient curve of the wing alone. The quantity $(\Delta\alpha)_v$ is found by estimating vortex positions and strengths and determining the average angle of attack induced by them and their images inside the body on the fins.

Equation (1) is limited to small angles. It can, however, be extended to higher angles by defining the equivalent angle of attack, α_{eq}, such that

$$\alpha_{eq} \equiv K_W \alpha_C + k_W \delta + (\Delta\alpha)_v \tag{2}$$

and rewriting Eq. (1) as

$$C_{N_{F(B)}} = C_{N_W}(\alpha_{eq}) \tag{3}$$

The success of Eq. (3) is demonstrated in Figs. 1 and 2 for two different fin planforms at a subsonic and a supersonic speed.[10] Equation (3) clearly correlates the data for different fin deflections. Wing-alone data[11] are also shown for comparison with the $M_\infty = 0.8$ results.

Although Eq. (2) allows us to account for different span-to-body diameter ratios and vortical flowfields through the K_W, k_W, and $(\Delta\alpha)_v$ factors, it is inadequate for missiles that are not in the "plus" attitude and for very high angles of attack. In the next section, a more general formula is derived.

Mathematical Development

We want the general derivation to include the effects of angle of attack, bank angle, fin deflection, body radius-to-semispan ratio, and vortical flowfield; that is, we want to find an α_{eq_i} for fin i such that

$$C_{N_W}(\alpha_{eq_i}) = C_{N_{F(B)_i}} [\alpha_C, \phi_i, \delta_{1-4}, a/s_m, (\Delta\alpha)_{v_i}] \tag{4}$$

For small angles of attack, zero bank angle ("plus" attitude), and undeflected vertical fins, Eq. (4) reduces to Eq. (1). However, for large angles of attack, a nonlinear definition of α_{eq} is required. Since there is no unique way to derive a nonlinear formula from the linear result, we are free to choose our approach provided that it is valid in certain limiting cases and reduces to the linear result in the limit of small angles. Our method is based on the use of average velocity components seen by the fin of interest. Those velocity components are put together to give α_{eq}.

Consider a cruciform wing-body combination in the "plus" attitude with the x axis rearward along the body axis, the y axis lateral along the right horizontal fin (fin 4), and the z axis

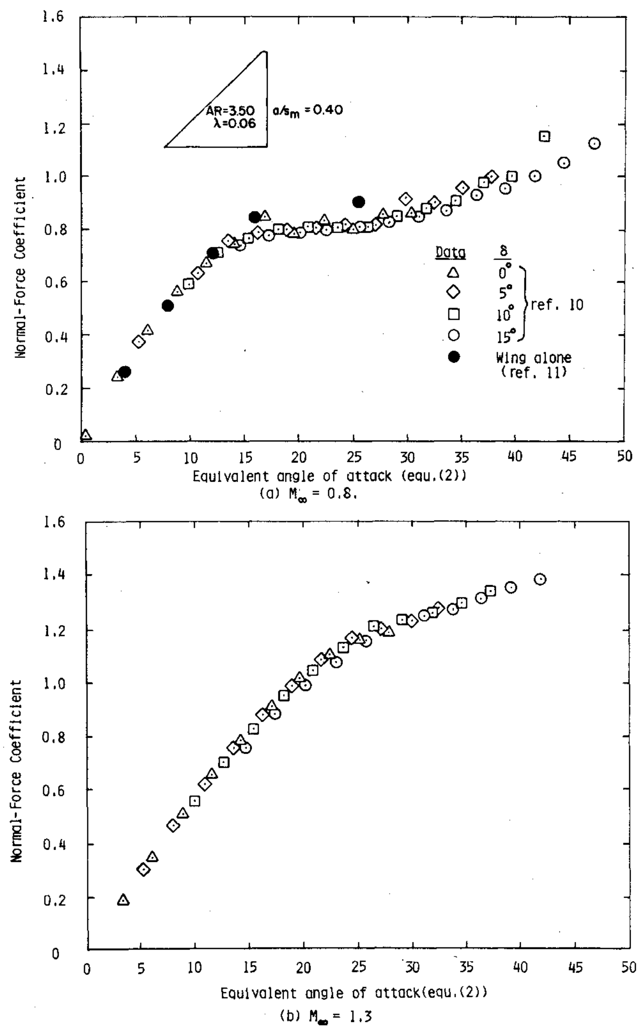

Fig. 1 Correlation of normal-force coefficient with equivalent angle of attack for moderate aspect ratio fin.

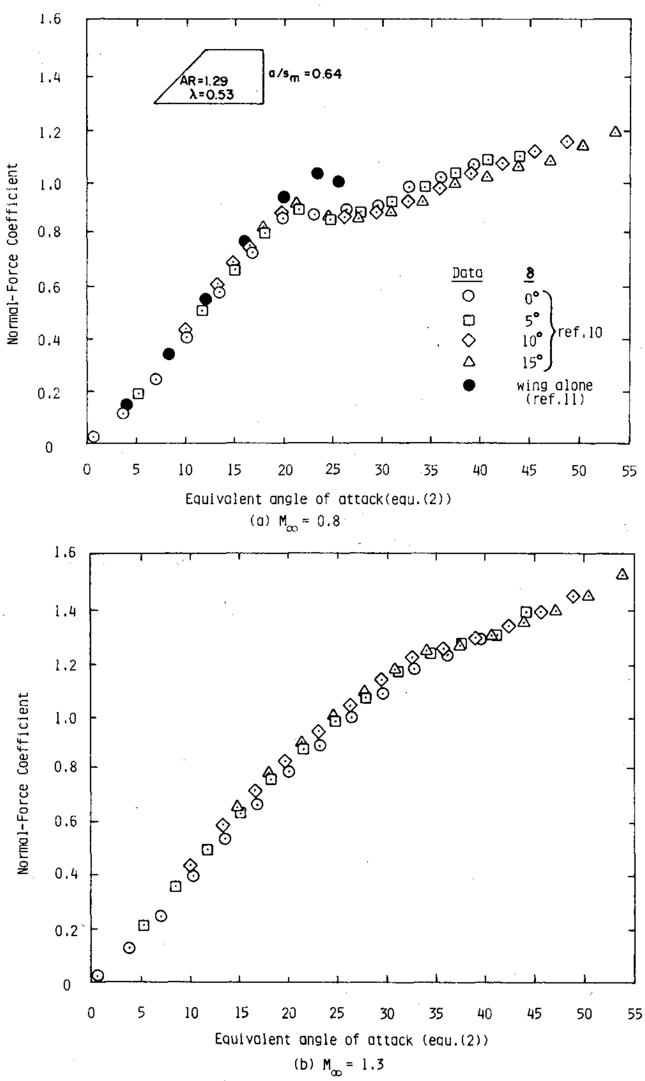

Fig. 2 Correlation of normal-force coefficient with equivalent angle of attack for low aspect ratio fin.

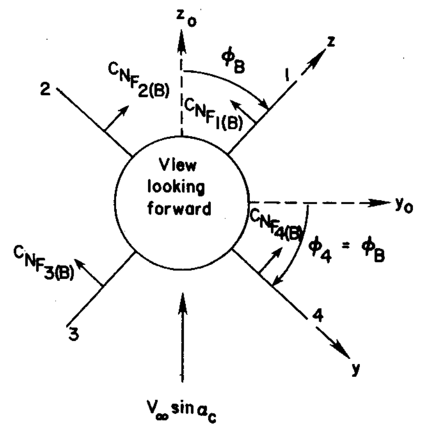

Fig. 3 Nomenclature for derivation of α_{eq} formulas. Fin deflection is positive leading edge toward leeward side of the body.

vertical (leeward) along the upper fin (fin 1). Let the combination first be pitched in a plane containing the freestream velocity and the x axis by an angle α_C. Let it then be rolled right wing down (windward) by angle ϕ_B. In a plane normal to the body axis, we now have the picture shown in Fig. 3.

We determine first the flowfield seen by the fins with no fin deflection and then consider the effects of rotating (deflecting) the fins in that flowfield. Referring to Fig. 3, we see that the components of the freestream velocity along x, y, and z are $V_\infty \cos\alpha_C$, $-V_\infty \sin\alpha_C \sin\phi_4$, and $V_\infty \sin\alpha_C \cos\phi_4$, respectively.‡ The component of freestream velocity normal to the plane of fin 4, V_{n_4}, is $V_\infty \sin\alpha_C \cos\phi_4$. Thus, the angle of attack induced on fin 4 *if it sees the freestream only* is

$$\tan\alpha_{eq_4} = V_{n_4} / V_{p_4} = \tan\alpha_C \cos\phi_4 \tag{5}$$

Equation (5) does not yet represent the actual angle of attack induced on fin 4 with no fins deflected because effects of the body, sideslip, and vortices have not been incorporated. We now consider these one at a time.

The presence of the body affects the flow in the cross-flow plane (wing-body interference). We account for this effect by multiplying V_{n_4} by the Beskin upwash factor K_W, that is,

$$V_{n_4} \Big|_{\text{freestream + body}} = K_W V_\infty \sin\alpha_C \cos\phi_4 \tag{6}$$

For favorable interference, $K_W > 1$. With increasing α_C and M_∞, K_W tends to decrease and become less than one.[13] We will assume that K_W is independent of ϕ as predicted by slender-body theory.

Any fin for which ϕ is not zero is sideslipping. Spreiter and Sacks[14] investigated this effect and found that the increment due to sideslip in the fin normal force was proportional to the product of α and β for the fin, i.e., $\alpha_C^2 \sin\phi_4 \cos\phi_4$. We use the same idea here to account for changes in V_{n_4} due to sideslip, that is,

$$V_{n_4} \Big|_{\text{sideslip}} = \frac{4}{AR} K_\phi V_\infty \alpha\beta = \frac{4}{AR} K_\phi V_\infty \sin^2\alpha_C \sin\phi_4 \cos\phi_4 \tag{7}$$

The introduction of the $4/AR$ coefficient makes the slender-body value of K_ϕ independent of aspect ratio.

To account for the effects of body vortices and vortices generated by upstream fins, we need a method to account for the equivalent angle of attack $(\Delta\alpha)_{v_i}$ induced on fin i by the vortical flowfield. By definition, we have

$$\left(\frac{V_{n_4}}{V_{p_4}} \right)_{\text{vortices}} = \tan(\Delta\alpha)_{v_4} \tag{8}$$

‡Note that we are using the sine definition of the angle of attack (see page 5 of Ref. 12).

Table 1 Slender body fin deflection factors (fin 4) for zero angle of attack[13]

a/s_m	Λ_{41}	Λ_{42}	Λ_{43}	Λ_{44}
0.0	−0.275	0.079	0.275	0.921
0.1	−0.230	0.073	0.230	0.890
0.2	−0.188	0.066	0.188	0.878
0.3	−0.149	0.057	0.149	0.879
0.4	−0.112	0.046	0.112	0.889
0.5	−0.078	0.034	0.078	0.905
0.6	−0.050	0.023	0.050	0.925
0.7	−0.027	0.013	0.027	0.946
0.8	−0.012	0.006	0.012	0.966
0.9	−0.003	0.001	0.003	0.984
1.0	0	0	0	1.000

If we assume that the vortices are rectilinear and parallel to the body axis, $(V_{p_4})_{\text{vortices}}$ is equal to the component of the freestream velocity which is parallel to the fin chord, $V_\infty \cos\alpha_C$, and we can write

$$V_{n_4} \Big|_{\text{vortices}} = V_\infty \cos\alpha_C \tan(\Delta\alpha)_{v_4} \tag{9}$$

Using relations (6),(7), and (9) and noting that V_{p4} is $V_\infty \cos\alpha_C$, we can define an equivalent angle of attack, $\hat{\alpha}_{eq_4}$, which includes body, sideslip, and vortex effects but no fin deflections, that is,

$$\tan\hat{\alpha}_{eq_4} = \frac{V_{n_4}|_{\text{freestream + body}} + V_{n_4}|_{\text{sideslip}} + V_{n_4}|_{\text{vortices}}}{V_{p_4}}$$

$$= K_W \tan\alpha_C \cos\phi_4 + \frac{4}{AR} K_\phi \tan\alpha_C \sin\alpha_C \sin\phi_4 \cos\phi_4 + \tan(\Delta\alpha)_{v_4} \tag{10}$$

Note that the sideslip and vortex terms in Eq. (10) cannot increase α_{eq_4} beyond 90 deg.

To account for the change in equivalent angle of attack due to fin deflection, we define a new quantity Λ_{ji} such that the effect on the angle of attack of fin i of the deflection of fin j is given by

$$(\Delta\alpha_{eq})_{ji} = \Lambda_{ji} \delta_j \tag{11}$$

where Λ_{ji} is given in Table 1 for slender-body theory for zero angle of attack. The final expression for the equivalent angle of attack of fin 4, which includes fin deflection, is§

$$\alpha_{eq_4} = \hat{\alpha}_{eq_4} + \sum_{j=1}^{4} \Lambda_{j4} \delta_j \tag{12}$$

On physical grounds we must use an angle addition theorem for fin deflection rather than a tangent addition theorem to allow for the possibility of α_{eq_4} exceeding 90 deg. In the definition of Λ_{ji} as incorporated in Eqs. (11) and (12), we have assumed implicitly that any vortex effects on fin i are not changed by deflecting the fins.

We can generalize Eqs. (10) and (12) by defining ϕ_4 to be the bank angle of fin i measured positive to windward from the y_0 axis. The generalized equations for fin i are

$$\tan\hat{\alpha}_{eq_i} = K_W \tan\alpha_C \cos\phi_i + \frac{4}{AR} K_\phi \tan\alpha_C \sin\alpha_C \sin\phi_i \cos\phi_i + \tan(\Delta\alpha)_{v_i} \tag{13}$$

§The zero angle-of-attack assumption built into Table 1 results in no effect of sideslip on $(\Delta\alpha_{eq})_{ji}$ if those factors are used. However, in general, Λ_{ji} is a function of ϕ.

and

$$\alpha_{eq_i} = \hat{\alpha}_{eq_i} + \sum_{j=1}^{4} \Lambda_{ji}\delta_j \tag{14}$$

Extension of Data Bases Using the α_{eq} Concept

Extension of Zero Bank Data

It has already been demonstrated in a previous section that a wing-alone data base can be used to estimate the normal-force coefficients of the horizontal fins on a circular body,

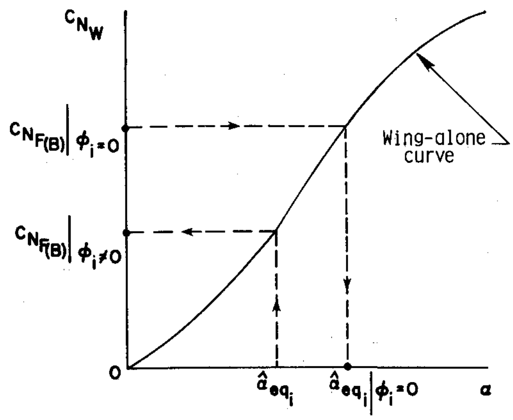

Fig. 4 Illustration of use of Eq. (17).

including effects of fin deflection. In this subsection we show how normal-force data for horizontal fins, together with the corresponding wing-alone curve, can be extended to any bank angle. For this discussion we will not include the effects of vortices or fin deflection.

For this case Eq. (3) reduces to

$$\tan\hat{\alpha}_{eq_i} = K_W \tan\alpha_C \left(\cos\phi_i + \frac{4}{AR}\frac{K_\phi}{K_W}\sin\alpha_C \sin\phi_i \cos\phi_i\right) \tag{15}$$

But for $\phi_i = 0$, we have

$$\tan\hat{\alpha}_{eq_i}\Big|_{\phi_i=0} = K_W \tan\alpha_C \tag{16}$$

Substituting Eq. (16) into Eq. (15) gives

$$\tan\hat{\alpha}_{eq_i} = \tan\hat{\alpha}_{eq_i}\Big|_{\phi_i=0}\left(\cos\phi_i + \frac{4}{AR}\frac{K_\phi}{K_W}\sin\alpha_C \sin\phi_i \cos\phi_i\right) \tag{17}$$

To use Eq. (16), one obtains $C_{N_{F(B)}}$ for $\phi_i = 0$ and the angle of attack of interest. Then $\hat{\alpha}_{eq}|_{\phi_i=0}$ is found from Eq. (3) and Eq. (17) is used to solve for $\hat{\alpha}_{eq_i}$. Then Eq. (3) is used again to find $C_{N_{F(B)}}$ for $\phi_i \neq 0$. The procedure is illustrated in Fig. 4.

An example of the accuracy of the method is given in Fig. 5 for aspect ratio 1 clipped delta fins with exposed semispan equal to the body radius. The Mach number is 2. The data came from the body-tail data base of Ref. 1 with the vortex

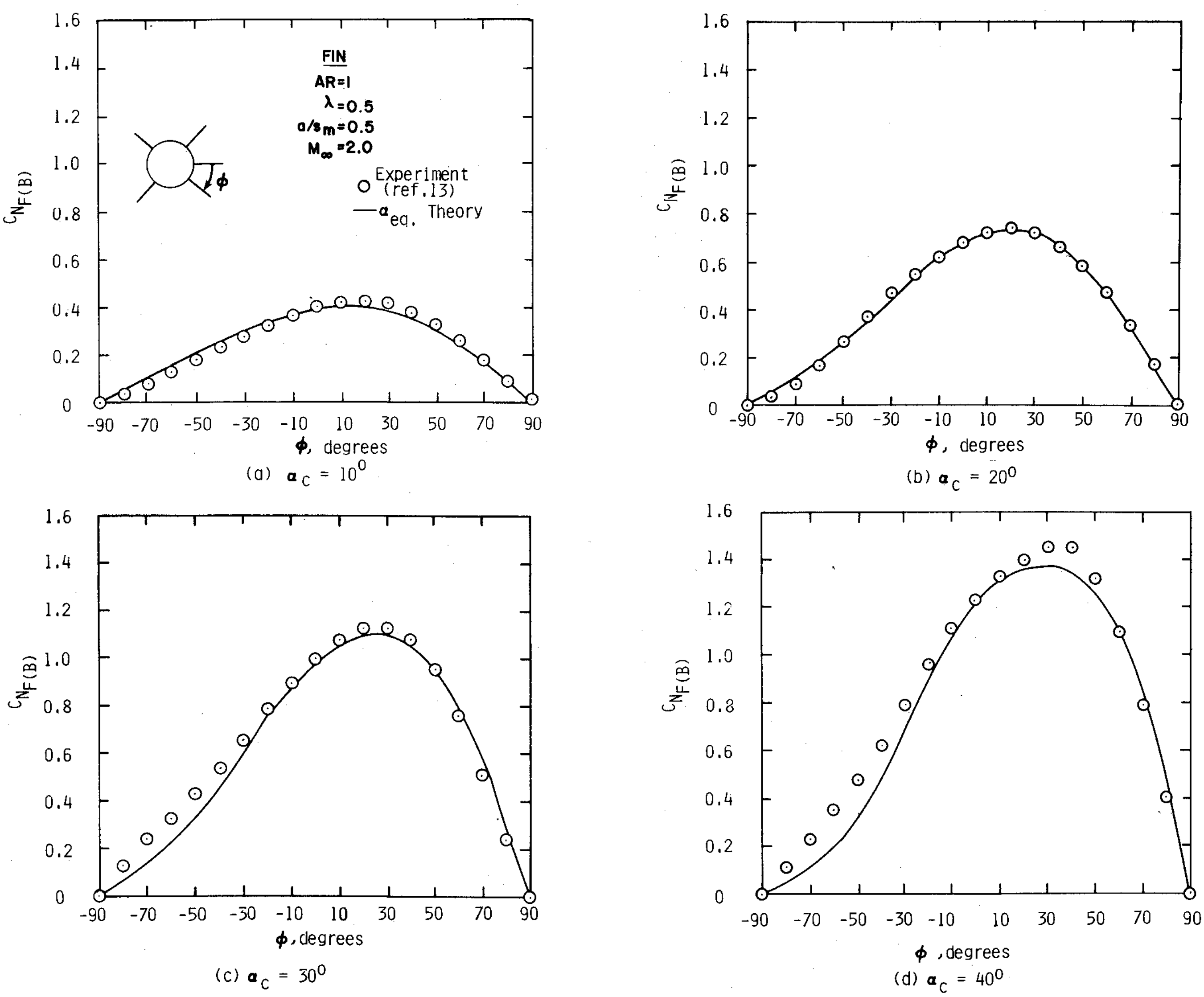

Fig. 5. Comparison of experiment with α_{eq} theory for extending low aspect ratio fin-on-body data base for $\phi = 0$ to include effects of roll.

effects removed.[13] Slender-body theory values of K_W and K_ϕ were used. The agreement is excellent except for the leeward bank angles at the higher angles of attack. We believe that the disagreement there is due to inaccuracies in the model used to extract the body vortex effects.

Additional examples of results for higher aspect ratio clipped delta fins are shown in Figs. 6 and 7. For these two cases, the K_ϕ/K_W ratios were chosen to give the best results for $\phi_i = \pm 45$ deg. Again the agreement is good. The fin of Fig. 7 is the same as that of Fig. 1. The data for Figs. 6 and 7 come from Refs. 15 and 10, respectively.

Spahr[15] and Hart[16] give comparisons similar to those shown above. Both used the linear version of Eq. (10) (small α_c limit). While Hart used Eq. (1) to get normal force, thereby sticking to the small-angle limit of the theory, Spahr used Eq. (3) together with the nonlinear wing-alone normal-force curve.

Note that we have not shown high angle-of-attack comparisons for subsonic Mach numbers. For this speed regime, stall effects are not handled particularly well by the method. This is probably because sideslip has a pronounced effect on stall characteristics.

If vortex effects cannot be neglected, Eq. (16) becomes

$$\tan\hat{\alpha}_{eq_i}\Big|_{\phi_i=0} = K_W \tan\alpha_C + \tan(\Delta\alpha)_{v_i}\Big|_{\phi_i=0} \tag{18}$$

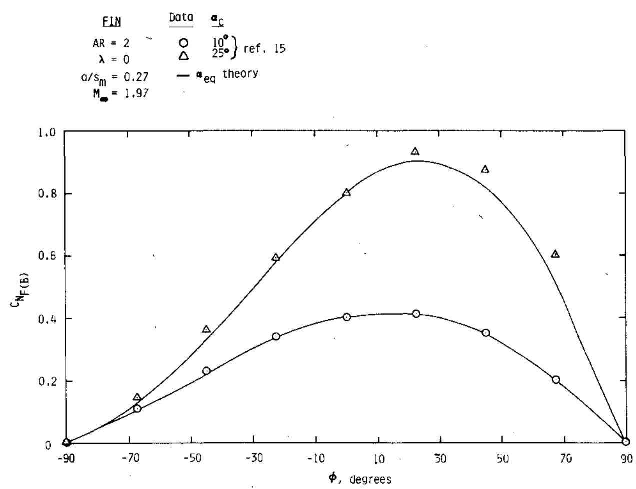

Fig. 6 Comparison of experiment with α_{eq} theory for moderate aspect ratio fins.

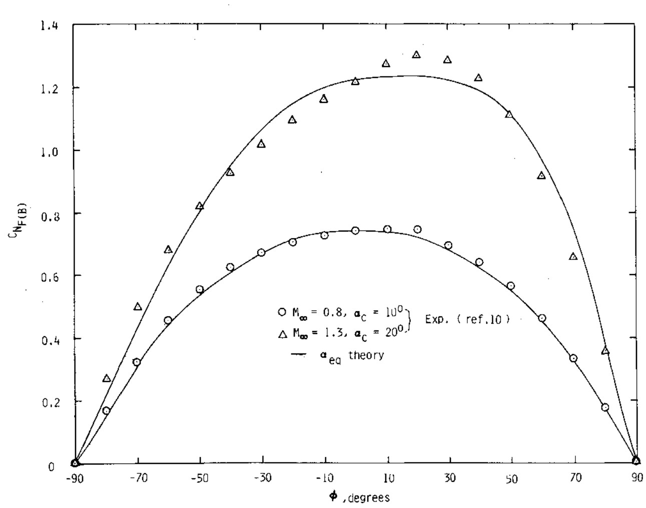

Fig. 7 Comparison of experiment with α_{eq} theory for high aspect ratio fins of Fig. 1.

Hence, Eq. (17) becomes

$$\tan\hat{\alpha}_{eq_i} = \left[\tan\hat{\alpha}_{eq_i} - \tan(\Delta\alpha)_{v_i}\right]_{\phi_i=0} \times\left[\cos\phi_i + \frac{4}{A\!\!\!R}\frac{K_\phi}{K_W}\sin\alpha_C\sin\phi_i\cos\phi_i\right] + \tan(\Delta\alpha)_{v_i} \tag{19}$$

The values of $(\Delta\alpha)_{v_i}$ must be estimated.

Correlation and Extension of Center-of-Pressure Data

The equivalent angle-of-attack concept also indicates that we can write equations for the fin center-of-pressure locations which are similar to Eq. (3) for the normal-force coefficient; that is,

$$\frac{\bar{x}_{F(B)}}{c_R} = \frac{\bar{x}_W(\alpha_{eq})}{c_R} \tag{20}$$

$$\frac{\bar{y}_{F(B)} - a}{s_m - a} = \frac{\bar{y}_W(\alpha_{eq})}{s} \tag{21}$$

Equations (20) and (21) taken together with Eq. (3) suggest that it should be possible to correlate center-of-pressure data as a function of fin normal-force coefficient; that is,

$$\bar{x} = \bar{x}(C_N) \tag{22}$$

$$\bar{y} = \bar{y}(C_N) \tag{23}$$

This result is a consequence of the α_{eq} concept.

An extensive demonstration of the axial center-of-pressure correlation for clipped delta fins is given in Ref. 17. A sampling of those results for a rectangular fin are given in Fig. 8 for subsonic and supersonic speeds. The fin-on-body results[1] for a given body angle of attack, α_C, are connected by a solid line. The wing-alone data are taken from Ref. 18.

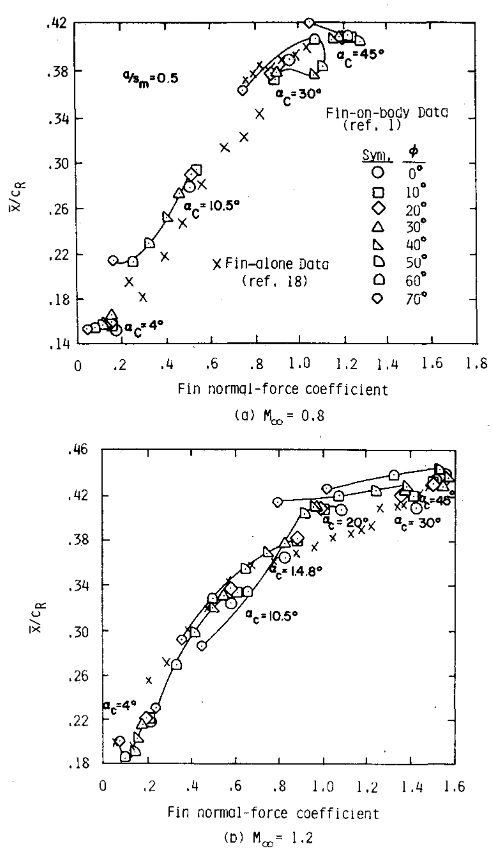

Fig. 8 Correlation of axial center-of-pressure positions for rectangular wing alone and fin-on-body at $\delta = 0$ deg; $A\!\!\!R = 1.0$.

The lateral center-of-pressure correlation was demonstrated in Ref. 13, samples of which are shown in Fig. 9. Note the expanded scale.

Extrapolation of Fin-on-Body Data Base

There are usually two situations that require extrapolation outside a fin-on-body data base (with roll), even if the fins have similar planforms: 1) the vortical flowfield seen by the fins is different from that encountered during the data base tests, and 2) the body radius-to-fin semispan ratio is different from that used in the tests.

Consider the first situation with a/s_m unchanged. Then writing Eq. (13) for two different cases and subtracting the results gives

$$\tan\hat{\alpha}_{eq_{i,2}} - \tan\hat{\alpha}_{eq_{i,1}} = \tan(\Delta\alpha)_{v_{i,2}} - \tan(\Delta\alpha)_{v_{i,1}} \tag{24}$$

If a method for estimating $(\Delta\alpha)_v$ is available, Eq. (24) together with Eqs. (14) and (3) give the change in normal-force coefficient due to the change in vortical flowfields.

Now consider the second situation with a/s_m and $(\Delta\alpha)_v$ both changed. Again we write Eq. (13) for two different cases and subtract to get

$$\tan\hat{\alpha}_{eq_{i,2}} - \tan\hat{\alpha}_{eq_{i,1}} = [K_{W_2} - K_{W_1}]\tan\alpha_C\cos\phi_i + \frac{4}{\mathcal{R}}[K_{\phi_2} - K_{\phi_1}]\tan\alpha_C\sin\alpha_C\sin\phi_i\cos\phi_i + \tan(\Delta\alpha)_{v_{i,2}} - \tan(\Delta\alpha)_{v_{i,1}} \tag{25}$$

As in the first situation, the change in vortical effects must be estimated, but now we must also estimate the changes in the K_W and K_ϕ effects.

It was shown in Ref. 13 that the Beskin upwash factor K_W can differ considerably from the slender-body theory values. Hence, the approach taken in that study was to use the data base as much as possible. Since K_W is nearly linear in a/s_m for slender-body theory, it was assumed that

$$K_W = A\frac{a}{s_m} + B \tag{26}$$

Since $K_W = 1$ for no body present ($a/s_m = 0$), we find that $B = 1$. The coefficient A can be found by evaluating K_W for the a/s_m of the data base using Eq. (18). A little algebra then gives

$$K_{W_2} - K_{W_1} \approx [K_{W_1} - 1]\left[\frac{(a/s_m)_2 - (a/s_m)_1}{(a/s_m)_1}\right] \tag{27}$$

Because of the $\tan\alpha_C\sin\alpha_C$ part of the K_ϕ term of Eq. (25), the sideslip term is usually considered to be second order especially for small values of α_C. However, for the low-to-moderate aspect ratios of typical missile fins, the term is not insignificant, especially for moderate-to-high values of α_C. For slender-body theory, K_ϕ is not linear with a/s_m, so the approach used for the K_W term will not work for the sideslip term. However, for moderate aspect ratios, it seems reasonable to simply use the slender-body theory values of K_ϕ for attached flow.[12]

For low aspect ratio fins, vortex shedding from the fin edges is a significant effect. Furthermore, studies of the shedding process show that the shedding is a strong function of the bank angle of the fin.[19] Until additional work is done to evaluate this effect, the best approach seems to be to evaluate K_{ϕ_1}, using Eq. (15) (once K_{W_1} has been found) at, say, $\phi_i = 45$ deg. Slender-body theory (SBT) could then be used to estimate K_{ϕ_2}. In mathematical terms, we would have

$$K_{\phi_2} - K_{\phi_1} = K_{\phi_1}\left(\frac{K_{\phi 2}}{K_{\phi 1}} - 1\right) \approx K_{\phi_1}\left[\left(\frac{K_{\phi 2}}{K_{\phi 1}}\right)_{\text{SBT}} - 1\right] \tag{28}$$

If any of the fins are deflected, then Eq. (14) must also be used. Writing Eq. (14) for the two different cases and subtracting the results gives

$$\alpha_{eq_{i,2}} - \alpha_{eq_{i,1}} = \hat{\alpha}_{eq_{i,2}} - \hat{\alpha}_{eq_{i,1}} + \sum_{j=1}^{4}(\Lambda_{ji,2} - \Lambda_{ji,1})\delta_j \tag{29}$$

Very little systematic data are available yet with which to determine the best approach for handling the term in parentheses in Eq. (29). However, since the slender-body theory values of Λ_{ji} are nearly linear in a/s_m, the approach used for the K_W term seems reasonable here.

Concluding Remarks

The equivalent angle-of-attack concept appears to be a very useful tool for use with both wing-alone and fin-on-body data bases. However, it is not limited to simple extrapolation from one configuration to another not-too-different configuration. Armed with accurate vortical flowfield models, one can even extrapolate body-tail data to configurations with two or more sets of fins.

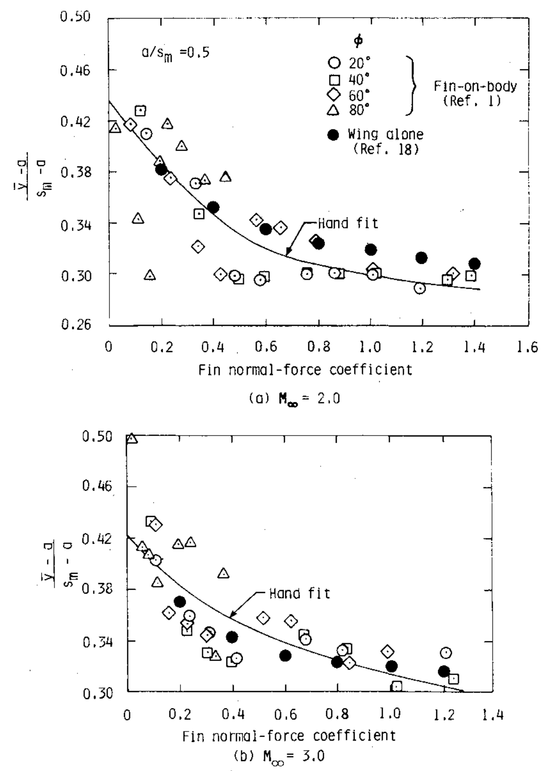

Fig. 9 Correlation of spanwise center-of-pressure positions for delta wings alone and fin-on-body at $\delta = 0$ deg; $\mathcal{R} = 1.0$.

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References

[1]Derrick, J., Spring, D., and Winn, G., "Aerodynamic Characteristics of a Series of Bodies With and Without Tails at Mach Numbers from 0.8 to 3.0 and Angles of Attack from 0° to 45°," Army Missile Command, TR-RD-7T-3, July 1976.

[2]Stallings, R.L. Jr. and Lamb, M., "Wing-Alone Aerodynamic Characteristics for High Angles of Attack at Supersonic Speeds," NASA TP 1889, July 1981.

[3]Briggs, M.M., Reed, R.E., and Nielsen, J.N., "Wing-Alone Aerodynamic Characteristics to High Angles of Attack at Subsonic and Supersonic Speeds," Nielsen Engineering & Research Inc., TR 269, Sept. 1982.

[4]Hemsch, M.J. and Nielsen, J.N., "Triservice/NASA/NEAR Program for Extending Missile Aerodynamic Data Base and Prediction Program Using Rational Modeling," Nielsen Engineering & Research, Inc., TR 282, Aug. 1982.

[5]Hemsch, M.J., Nielsen, J.N., Smith, C.A., and Perkins, S.C. Jr., "Component Aerodynamic Characteristics of Banked Cruciform Missiles with Arbitrary Control Deflection," AIAA Paper 77-1153, Aug. 1977.

[6]Smith, C.A., Nielsen, J.N., and Hemsch, M.J., "Prediction of Aerodynamic Characteristics of Cruciform Missiles to High Angles of Attack," AIAA Paper 79-0024, Aug. 1979.

[7]Mendenhall, M.R., "Predicted Vortex Shedding from Noncircular Bodies in Supersonic Flow," *Journal of Spacecraft and Rockets,* Vol. 18, Sept./Oct. 1981, pp. 385-392.

[8]Allen, J.M. and Dillenius, M.F.E., "Vortex Development on Slender Missiles at Supersonic Speeds," *Journal of Spacecraft and Rockets,* Vol. 17, July-Aug. 1980, pp. 377-378.

[9]Pitts, W.C., Nielsen, J.N., and Kaattari, G.E., "Lift and Center of Pressure of Wing-Body-Tail Combinations at Subsonic, Transonic, and Supersonic Speeds," NACA Rept. 1307, 1957.

[10]Hemsch, M.J. and Nielsen, J.N., "Test Report for Canard Missile Tests in Ames 6- by 6-Foot Supersonic Wind Tunnel," Nielsen Engineering & Research, Inc., TR 72, Aug. 1974.

[11]Emerson, H.F., "Wind-Tunnel Investigation of the Effects of Clipping the Tips of Triangular Wings of Different Thickness, Camber, and Aspect Ratio—Transonic Bump Method," NACA TN 3671, June 1956.

[12]Nielsen, J.N., *Missile Aerodynamics,* McGraw-Hill Book Co., New York, 1960.

[13]Nielsen, J.N., Hemsch, M.J., and Smith, C.A., "A Preliminary Method for Calculating the Aerodynamic Characteristics of Cruciform Missiles to High Angles of Attack Including Effects of Roll Angle and Control Deflections," Office of Naval Research, CR215-226-4F, Nov. 1977.

[14]Spreiter, J.R. and Sacks, A.H., "A Theoretical Study of the Aerodynamics of Slender Cruciform-Wing Arrangements and Their Wakes," NACA Rept. 1296, 1957.

[15]Spahr, J.R., "Contribution of the Wing Panels to the Forces and Moments of Supersonic Wing-Body Combinations at Combined Angles," NACA TN 4146, Jan. 1958.

[16]Hart, H.H., "Hypersonic Delta-Wing-Body Interference," Paper presented at the Seventh U.S. Navy Symposium on Aeroballistics, June 1966.

[17]Nielsen, J.N. and Goodwin, F.K., "Preliminary Method for Estimating Hinge Moments of All-Movable Controls," Nielsen Engineering & Research, Inc., TR 268, March 1982.

[18]Baker, W.B., Jr., "Static Aerodynamic Characteristics of a Series of Generalized Slender Bodies With and Without Fins at Mach Numbers from 0.6 to 3.0 and Angles of Attack from 0 to 180 Degrees," Arnold Engineering Development Center, TR-75-124, Vols. I and II, May 1976.

[19]Telste, J.G. and Lugt, H.J., "Vortex Shedding from Finned Circular Cylinders," David Taylor Naval Ship Research and Development Center, Rept. 80/124, Nov. 1980.